

Experimental and Numerical Study of Solidification Process in Unsaturated Granular Packed Bed

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The solidification process in unsaturated granular packed beds is investigated theoretically and numerically. The water transport to the solidification interface, that is, freezing front, plays an important role on solidification in unsaturated granular packed beds. It is found that the rate of the absorption of water into the frozen layer depends on the freezing heat flux and the water saturation at the solidification interface. As a result, ice content in the frozen layer is related to the rate of the absorption of water and the freezing heat flux. The one-dimensional solidification model associated with phase change conditions and the coordinate transformation techniques for the moving boundary problem are completely presented to calculate the temperature and water saturation distributions and the interface position. Later, the predicted results are compared with experimental results obtained using granular packed beds composed of glass beads and water.

Nomenclature

c_p	=	specific heat at constant pressure, J/kgK
d	=	particle size, m
dW/dt	=	rate of absorption of water, kg/m ² s
f_w	=	water flow rate, kg/m ² s
h	=	heat transfer coefficient, W/m ² K
K	=	permeability, m ²
L	=	latent heat, J/kg
p	=	pressure, Pa
q_f	=	freezing heat flux, J/m ² s
R	=	position of solidification interface, m
s	=	water or ice saturation
s_{ir}	=	irreducible saturation
T	=	temperature, °C
t	=	time, s
u	=	velocity, m/s
x	=	horizontal direction coordinate, m
ε	=	porosity
η, ζ	=	coordinate transformation
λ	=	effective thermal conductivity, W/mK
ρ	=	density, kg/m ³

Subscripts

b	=	brine or solidification interface
c	=	capillary
e	=	effective
i	=	ice
p	=	particle
r	=	relative
s	=	surface of saturation
w	=	water or wet
x	=	x component
0	=	initial, reference

I. Introduction

THE phenomenon of solidification or freezing process in unsaturated porous media is widely encountered in nature and

in many engineering systems. Some of the specific applications include pipeline transport in permafrost regions and cryosurgery, as well as in the transportation of coal in cold weather, ice accretion on vehicles and static structures, solidification of alloys, food processing, chemical processes, cryopreservation of engineering tissues, and many others.

A similar problem, that is, the simultaneous heat and mass transfer problem in porous media based on Whitaker's theory, has been studied by many authors including Whitaker and Chou,¹ Plumb et al.,² Udell,³ Kaviany and Mittal,⁴ Rogers and Kaviany,⁵ and Rattanadecho et al.^{6–8}

Up to the present time, the related problem of solidification in porous media in the absence of an unsaturated state has been investigated both experimentally and numerically by many researchers and up to date reviews are available: Hashemi and Sliepcevich,⁹ Frivik and Comini,¹⁰ Sparrow and Broadbent,¹¹ Weaver and Viskanta,¹² and Chellaiah and Viskanta.¹³

Only a very limited amount of analytical and experimental work on phase change heat transfer in unsaturated porous media has been reported, and understanding of the phenomenon is incomplete. At macroscopic level, there exist four distinct phases (porous matrix, ice, water, and air) within any representative elementary volume of the unsaturated porous media system.

When solving a moving boundary problem, that is, solidification process of unsaturated porous media, complications arise due to the motion of the solidification interface with the phase transformation and the absorption of water at this interface. As such, the position of the interface is not known a priori, and the domains over which the energy equations are solved vary. There exists a discontinuity in the temperature gradient, as well as liquid saturation gradient at the solidification interface. Furthermore, during solidification in unsaturated porous media, the mechanism of the water transport to the interface and the growth of the segregated ice must be clearly investigated. Namely, the water in the unfrozen layer is absorbed to the interface due to capillary and osmotic actions so that ice content in the frozen layer increases, and a segregated ice layer often grows between frozen and unfrozen layers. In this paper, the first systematical study on solidification process in unsaturated porous media associated with water transport to the interface is described.

The purpose of this study is to report on theoretical and experimental studies of the solidification process in unsaturated porous media, that is, a granular packed bed at a fundamental level. The results of numerical calculations and a comparison of experimental data with predictions for the temperature distribution, the water saturation distributions, and the position of solidification interface based on the one-dimensional model are reported. It is hoped that this study will enhance physical understanding of the solidification process in unsaturated porous media such as artificial freezing of

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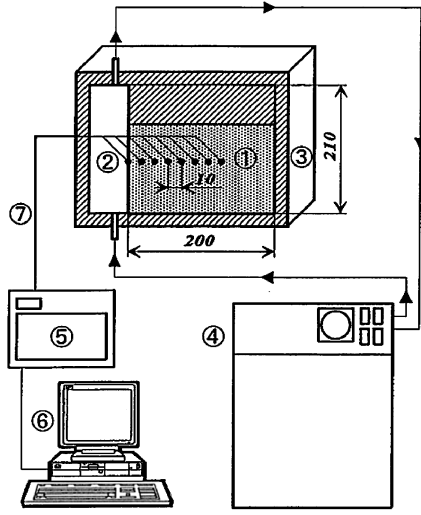


Fig. 1 Experimental apparatus: 1) packed bed, 2) cooling heat exchanger, 3) insulator, 4) cooling tank with refrigerator, 5) recorder, 6) computer, and 7) thermocouples.

ground for mining and construction purposes, ablation of thermal shields, degradation of permafrost, and purposeful ground freezing during geotechnical endeavors.

II. Experimental Apparatus

Figure 1 shows the apparatus where solidification experiments are performed, a rectangular test cell with inside dimensions of 130-mm height, 110-mm length, and 50-mm depth. The horizontal top and bottom walls and the vertical front and back walls are made of acrylic resin. The entire test cell is covered with 60-mm-thick Styrofoam on all sides to minimize the effect of heat losses and condensation of moisture at the walls. The test cavity is filled with a mixture of water and uniform size spherical glass beads with a diameter of 0.15 mm (porous matrix). The vertical front wall, which serves as the heat source, is a multipass heat exchanger. The heat exchanger is connected through a valve system to a constant temperature bath, where an ethylene glycol–water solution is used as the cooling medium.

Throughout the experiments, the test cell is set up in a constant room temperature held at 10°. Measurement of temperature distributions inside the test cell is made with 10 Cu–Co thermocouples with diameter of 0.2 mm. All thermocouples are positioned such that the temperatures are measured along the horizontal centerplane of the test cell at each 10-mm interval. The thermocouples are connected to the data logger and a computer through which the temperatures could be measured and store at preselected time intervals. The position of solidification interface in the packed bed is determined by interpolating the fusion temperature from the thermocouple reading.

The water or ice saturations in the packed bed are defined as the fractions of the volume occupied by water or ice to the volume of the pores. They are obtained by weighing the dry and wet mass of the sample, which is cut out in volume of about 10 mm³ at the end of each run.

The water saturation formula can be described in the following form:

$$s = \frac{\rho_p(1 - \varepsilon)(m_w - m_d)}{\rho_w \varepsilon m_d} \quad (1)$$

where m_w and m_d are the wet and dry masses of the sample, respectively.

During the solidification experiments, the uncertainty of the data might be due the variations in humidity, room temperature, and human errors. The calculated uncertainties in all tests are less than 2.85%. Additionally, a possible error in the predicted results might be due to the uncertainties in the calculated effective thermal conductivity and the permeability of the porous media.

III. Analysis of Water Transport Due to Solidification

Figure 2 shows typical profiles of ice and water saturations during the solidification process in an unsaturated granular packed bed. It is observed that the profiles are not uniform, that is, ice saturation is increased and water saturation is decreased as compared with the initial state. This means the water in the unfrozen layer is absorbed to the solidification interface or freezing front during the solidification process. From a macroscopic point of view, the water transport associated with phase change conditions due to solidification is discussed.

Figure 3 shows the typical moisture characteristic curve (the relationship between capillary pressure p_c and water saturation s) for the different particle sizes obtained from previous experiments.⁸ In the case of the same capillary pressure, a small particle size corresponds to higher water saturation. Now the case where two packed beds having the same water saturation and different particle sizes at the interface are adjusted as shown in Fig. 4 is considered. Because the capillary pressure has the same value at the interface between two beds, the water should be moved from the coarse bed to the fine bed. The water transport due to solidification may be considered as similar to this phenomenon. Namely, the freezing of the water around each particle in unsaturated packed bed allows gaps between pores to be narrower so that, to keep the same capillary pressure at the solidification interface, the water in the unfrozen layer should be absorbed to the interface. In this study, we obtain the rate of the absorption of water by the following procedure.

Consider an energy balance at the interface or freezing front. The motion of the frozen layer is then given by

$$\rho_i L \varepsilon s_{ib} \frac{dR}{dt} = \lambda_i \left. \frac{\partial T}{\partial x} \right|_{x=R} - \lambda_w \left. \frac{\partial T}{\partial x} \right|_{x=R} = q_f \quad (2)$$

where λ is the effective thermal conductivity based on the assumption of thermal equilibrium between phase change material (water or ice) and the porous matrix (glass beads) and depends on water

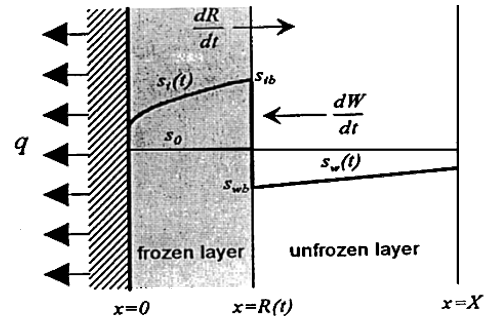


Fig. 2 Schematic profile of ice and water saturations due to solidification.

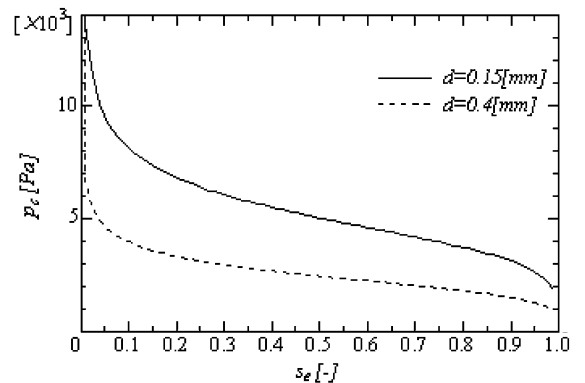


Fig. 3 Typical relationship between capillary pressure and water saturation.

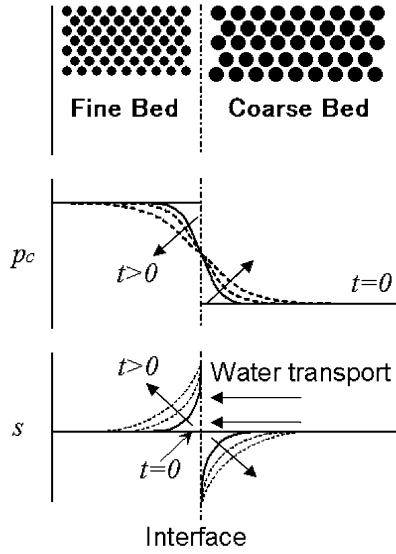


Fig. 4 Schematic diagram showing water transport at the interface between different porous beds.

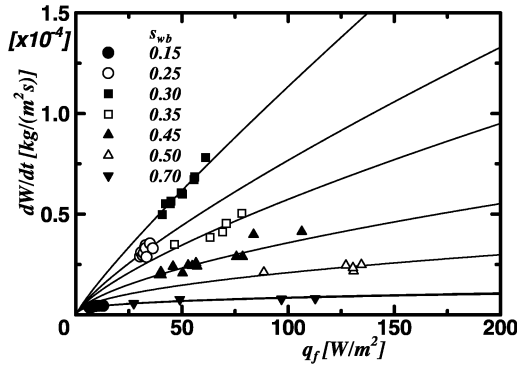


Fig. 5 Relationship between the rate of the absorption of the water and the freezing heat flux.

or ice saturation. The right-hand term in Eq. (2) is the heat flux of freezing. The rate of the absorption of water, which is related to the motion of frozen layer and the water and ice saturations at the interface, is given by

$$\frac{dW}{dt} = \varepsilon(\rho_i s_{ib} - \rho_w s_{wb}) \frac{dR}{dt} \quad (3)$$

Equate Eqs. (2) and (3), which yields the following equation:

$$\frac{dW}{dt} = \frac{(\rho_i s_{ib} - \rho_w s_{wb})}{\rho_i L s_{ib}} \left(\lambda_i \left. \frac{\partial T}{\partial x} \right|_{x=R} - \lambda_w \left. \frac{\partial T}{\partial x} \right|_{x=R} \right) \quad (4)$$

where the rate of the absorption of water due to solidification is obtained from measuring the heat flux of freezing and the water and ice saturations at the interface.

Figure 5 shows the relationship between the rate of the absorption of water (dW/dt), and the freezing heat flux q_f as a parameter of water saturation at the interface, s_{wb} . The rate of the absorption of water is increased with increasing in freezing heat flux. This is because a higher freezing heat flux leads to a faster change of porous structure due to solidification, that is, it leads to a larger water saturation difference at the interface between the frozen and unfrozen layers to keep the same capillary pressure. Also, the rate of the absorption of water depends on water saturation at the interface; however, the relationship is more complex. Figure 6, based on Fig. 5, shows the effect of s_{wb} on dW/dt . It is found that the rate of the absorption of water takes the maximum value near the water saturation of 0.3 and

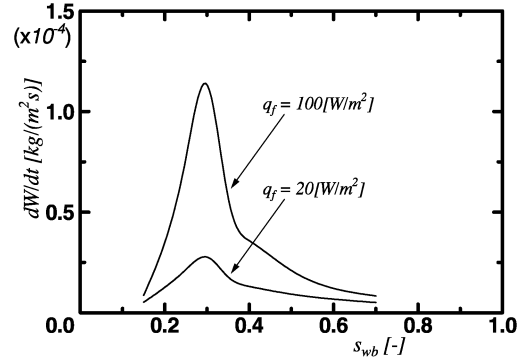


Fig. 6 Relationship between the rate of the absorption water and the water saturation at the interface.

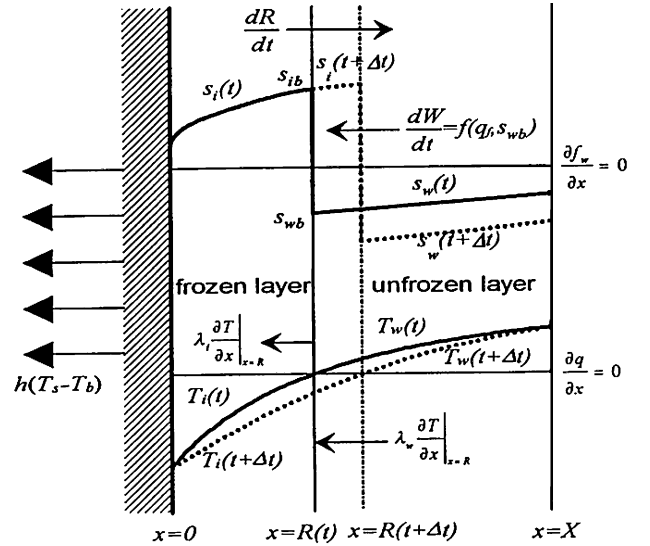


Fig. 7 Physical model for solidification process.

it decreases beyond or below its saturation. This may be explained by considering that the difference in water saturation among particle sizes becomes the most remarkable near the water saturation of 0.3, as shown in Fig. 3.

IV. Analysis of Solidification Process in Unsaturated Granular Packed Bed

In Fig. 7, the modeled physical system consists of two layers, namely, frozen layer and unfrozen layer. Inside the frozen layer, only heat transport takes place, whereas inside the unfrozen layer both heat and water transport can occur. Initially, the system is at a uniform temperature greater than or equal to the fusion temperature of a liquid. At time $t > 0$, a uniform temperature less than the fusion temperature is imposed on the left wall. Solidification is initiated at this wall, and the solidification interface moves from left to right. The following simplifying assumptions are made in the analysis:

1) The one-dimensional analysis for solidification in unsaturated granular packed bed is performed. The solidification interface is a planar due to the good control of the imposed temperature gradient in the solidification experiment.

2) When the energy balance equation (2) is considered at the solidification interface, it predicts the position of solidification interface and is based on an energy balance of an infinitesimal control volume along the planar interface.

3) The porous medium is isotropic and homogeneous and has uniform porosity. Therefore, the volume average model for isotropic and homogeneous material can be used in the theoretical modeling and analysis.

4) The water flow in the unfrozen layer is governed by Darcy's law.

- 5) The volumetric change due to solidification is negligible.
- 6) The natural convection is absent.
- 7) The frozen–unfrozen interface is clearly defined, that is, the phase-change material has a well-defined fusion temperature.
- 8) There is thermal equilibrium between the phase-change material and the porous matrix. This may be the case when the porous matrix has a slightly larger thermal conductivity than the phase-change material, and the interphase heat transfer can be properly neglected.

Water Transport Equation

The water conservation equation in the unfrozen layer based on Darcy's law is given as

$$\rho_w \frac{\partial s}{\partial t} = -\frac{\partial f_w}{\partial x} = -\frac{\partial}{\partial x}[\rho_w u_w]$$

or in another form,

$$\rho_w \frac{\partial s}{\partial t} = -\frac{\partial f_w}{\partial x} = -\rho_w \frac{\partial}{\partial x} \left[\frac{K K_{rw}}{\mu_w} \frac{\partial p_c}{\partial x} - \rho_w g \right] \quad (5)$$

The system of the conservation equations obtained for the multi-phase transport mode requires constitutive equations for relative permeabilities K_{rw} , capillary pressure p_c , and capillary pressure functions (Leverett functions) $J(s_e)$. A typical set of constitutive relationships for the liquid and gas system is given by

$$K_{rw} = s_e^3 \quad (6)$$

where s_e is the effective water saturation associated with the irreducible water saturation s_{ir} and is defined by

$$s_e = (s - s_{ir}) / (1 - s_{ir}) \quad (7)$$

The capillary pressure p_c is further assumed to be adequately represented by Leverett's well-known $J(s_e)$ functions. The relationship between the capillary pressure and the water saturation is defined by using Leverett functions $J(s_e)$,

$$p_c = (\sigma / \sqrt{K/\varepsilon}) J(s_e) \quad (8)$$

where the Leverett functions $J(s_e)$ is represented by (see Ratanadecho et al.⁸)

$$J(s_e) = 0.325[1/s_e - 1]^{0.217} \quad (9)$$

Energy Conservation Equations

The energy conservation equations in the frozen and unfrozen layers are given, respectively, by

$$\frac{\partial}{\partial t}[(\rho c_p)_{Ti} T_i] = \frac{\partial}{\partial x} \left[\lambda_i \frac{\partial T_i}{\partial x} \right] \quad (10)$$

$$\frac{\partial}{\partial t}[(\rho c_p)_{Tw} T_w] = \frac{\partial}{\partial x} \left[\lambda_w \frac{\partial T_w}{\partial x} \right] - \frac{\partial (c_{pw} f_w T_w)}{\partial x} \quad (11)$$

where $(\rho c_p)_T$ is the effective heat capacitance of the water, ice, and matrix mixture and λ is the effective thermal conductivity depending on water or ice saturation. Under the thermal equilibrium condition, the effective heat capacity is given as

$$(\rho c_p)_{T,j} = \rho_j c_{p,j} \varepsilon s + \rho_p c_{pp} (1 - \varepsilon), \quad j = i, w \quad (12)$$

where the subscript j represents either the frozen layer or the unfrozen layer. Based on the experimental results by Kiyohashi et al.,¹⁴ the effective thermal conductivity λ is represented as a function of water or ice saturation:

$$\lambda_j = \lambda_{\max,j} / [1 + A_j \exp(-5.95s_j)], \quad j = i, w \quad (13)$$

Table 1 Constant values used in Eq. (16)

s_{wb}	C	D
0.15	2.0	0.32
0.25	2.0	0.79
0.30	2.0	0.88
0.35	2.0	0.73
0.45	2.0	0.63
0.50	2.0	0.51
0.70	2.0	0.31

where constant A and $\lambda_{\max}$ corresponding to the unfrozen and frozen layers are

$$A_i = 7.06, \quad A_w = 3.78, \quad \lambda_{\max,i} = 1.35, \quad \lambda_{\max,w} = 0.8 \quad (14)$$

The motion of frozen layer or solidification interface is represented by

$$\rho_i L \varepsilon s_{ib} \frac{dR}{dt} = \lambda_i \left. \frac{\partial T}{\partial x} \right|_{x=R} - \lambda_w \left. \frac{\partial T}{\partial x} \right|_{x=R} \quad (15)$$

Note that the ice saturation at the interface, s_{ib} depends on the rate of the absorption of water, and the freezing heat flux, and the water saturation at the interface, s_{wb} , and is approximated by the following equation, which is obtained by applying spline interpolation to the experimental results as shown in Fig. 5:

$$\frac{dW}{dt} = \varepsilon (\rho_i s_{ib} - \rho_w s_{wb}) \frac{dR}{dt} = C q_f^{D(s_{wb})} \quad (16)$$

where the constant values of C and D for different values of s_{wb} used in Eq. (16) are shown in Table 1.

Equate Eqs. (15) and (16), which yields the following equation:

$$s_{ib} = s_{wb} / \left(\frac{\rho_i}{\rho_w} \left[1 - \left(L \frac{dW}{dt} / q_f \right) \right] \right) = s_{wb} / \left(\frac{\rho_i}{\rho_w} \left(1 - \frac{L C q_f^D}{q_f} \right) \right) \quad (17)$$

The other boundary conditions and the initial conditions are

$$\begin{aligned} t = 0, \quad x \geq 0 : \quad & s_i = s_0 \\ s_w = s_0, \quad T_i = 0, \quad T_w = T_\infty \\ t > 0, \quad x = 0 : \quad & q = h(T_s - T_b) \\ t > 0, \quad x = R_0 : \quad & \frac{\partial f_w}{\partial x} = 0, \quad \frac{\partial q}{\partial x} = 0 \end{aligned} \quad (18)$$

V. Coordinate Transformation

In solving a moving boundary problem including phase change, complications arise due to the motion of the solidification interface. In this study, the governing equations of water and heat transport were solved by using a coordinate transformation technique based on a boundary fixing method coupled with implicit time schemes.¹⁵ The detailed coordinate transformation technique is presented as follows.

In Frozen Layer

For the coordinate transformation in the frozen layer, we use the following coordinate system:

$$\eta = x/R(t), \quad 0 \leq x \leq R(t) \quad (19)$$

where $R(t)$ is distance between the cooling surface and solidification interface. With use of the coordinate transformation technique, the physical space (x, t) is then transformed to the mapping space

$[\eta(x, t), t]$. The differential operators with the coordinate transformation are mathematically related to the following equations:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \eta} \frac{\partial \eta}{\partial x} = \frac{1}{R} \frac{\partial}{\partial \eta} \quad (20)$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \eta} \frac{\partial \eta}{\partial t} + \frac{\partial}{\partial t} = \frac{\partial}{\partial t} - \frac{\eta \dot{R}}{R} \frac{\partial}{\partial \eta} \quad (21)$$

where $\dot{R}$ is the time derivative of the solidification interface. According to Eqs. (20) and (21), the energy equation in the frozen layer [Eq. (10)] can be then transformed to the following equation:

$$\frac{\partial}{\partial t}[(\rho c_p)_{T,i} T_i] = \frac{\eta \dot{R}}{R} \frac{\partial}{\partial \eta}[(\rho c_p)_{T,i} T_i] + \frac{1}{R^2} \frac{\partial}{\partial \eta} \left[\lambda_i \frac{\partial T_i}{\partial \eta} \right] \quad (22)$$

In Unfrozen Layer

For the coordinate transformation in the unfrozen layer, we use the following coordinate system:

$$\zeta = \frac{x - R(t)}{R_0 - R(t)}, \quad R(t) \leq x \leq R_0 \quad (23)$$

The differential operators with coordinate transformation are also mathematically related to the following equations:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \zeta} \frac{\partial \zeta}{\partial x} = \frac{1}{R_0 - R} \frac{\partial}{\partial \zeta} \quad (24)$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \zeta} \frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial t} = \frac{\partial}{\partial t} - \frac{\dot{R}(1 - \zeta)}{(R_0 - R)} \frac{\partial}{\partial \zeta} \quad (25)$$

According to Eqs. (24) and (25), the water transport and energy equations in the unfrozen layer [Eq. (11)] can be transformed to the following equations, respectively:

$$\varepsilon \frac{\partial s}{\partial t} = \varepsilon \frac{\dot{R}(1 - \zeta)}{R_0 - R} \frac{\partial s}{\partial \zeta} - \frac{1}{(R_0 - R)^2} \frac{\partial}{\partial \zeta} \left[\frac{K K_{rw}}{\mu_w} \left(\frac{\partial p_c}{\partial \zeta} \right) \right] \quad (26)$$

$$\begin{aligned} \frac{\partial}{\partial t}[(\rho c_p)_{T,w} T_w] &= \frac{\dot{R}(1 - \zeta)}{(R_0 - R)} \frac{\partial}{\partial \zeta}[(\rho c_p)_{T,w} T_w] \\ &- \frac{1}{R_0 - R} \frac{\partial}{\partial \zeta} [c_{pw} f_w T_w] + \frac{1}{(R_0 - R)^2} \frac{\partial}{\partial \zeta} \left[\lambda_w \frac{\partial T_w}{\partial \zeta} \right] \end{aligned} \quad (27)$$

At Solidification Interface

The motion of solidification interface [Eq. (15)] can be also transformed to the following equation:

$$\rho_i L \varepsilon s_{ib} \frac{dR}{dt} = \frac{\lambda_i}{R} \frac{\partial T}{\partial \eta} \Big|_{x=R} - \frac{\lambda_w}{R_0 - R} \frac{\partial T}{\partial \zeta} \Big|_{x=R} \quad (28)$$

VI. Numerical Schemes

In this study, the method of finite differences based on the notion of the control volumes¹⁶ is used. The generalized system of the nonlinear equations (22) and (26–28) is integrated over typical control volumes. After integration over each control volume within computational mesh, a system of nonlinear equations results whereby each equation can be cast into a numerical discretization of the generalized conservation equation.

Heat transport equation in frozen layer:

$$\begin{aligned} &\frac{(\rho c_p)_{T,i,i}^{n+1} T_{i,i}^{n+1} - (\rho c_p)_{T,i,i}^n T_{i,i}^n}{\Delta t} \\ &= \frac{\eta \dot{R}}{R} \frac{(\rho c_p)_{T,i,i+1/2}^{n+1} T_{i,i+1/2}^{n+1} - (\rho c_p)_{T,i,i-1/2}^{n+1} T_{i,i-1/2}^{n+1}}{\Delta \eta} \\ &+ \frac{1}{R} \frac{1}{\Delta \eta} \left[\lambda_{i,i+1/2}^{n+1} \left(\frac{T_{i,i+1}^{n+1} - T_{i,i}^{n+1}}{R \Delta \eta} \right) \right. \\ &\quad \left. - \lambda_{i,i-1/2}^{n+1} \left(\frac{T_{i,i}^{n+1} - T_{i,i-1}^{n+1}}{R \Delta \eta} \right) \right] \end{aligned} \quad (29)$$

Water and heat transport equations in unfrozen layer:

$$\begin{aligned} \varepsilon \frac{s_i^{n+1} - s_i^n}{\Delta t} &= \varepsilon \frac{\dot{R}(1 - \zeta) s_{i+1/2}^{n+1} - s_{i-1/2}^{n+1}}{\Delta \zeta} \\ &- \frac{1}{(R_0 - R)} \frac{1}{\Delta \zeta} \left[\frac{K K_{rw,i+1/2}^{n+1}}{\mu_w} \left(\frac{p_{c,i+1}^{n+1} - p_{c,i}^{n+1}}{(R_0 - R) \Delta \zeta} \right) \right. \\ &\quad \left. - \frac{K K_{rw,i-1/2}^{n+1}}{\mu_w} \left(\frac{p_{c,i}^{n+1} - p_{c,i-1}^{n+1}}{(R_0 - R) \Delta \zeta} \right) \right] \end{aligned} \quad (30)$$

$$\begin{aligned} &\frac{(\rho c_p)_{T,w,i}^{n+1} T_{w,i}^{n+1} - (\rho c_p)_{T,w,i}^n T_{w,i}^n}{\Delta t} \\ &= \frac{\dot{R}(1 - \zeta)}{R_0 - R} \frac{(\rho c_p)_{T,w,i+1/2}^{n+1} T_{w,i+1/2}^{n+1} - (\rho c_p)_{T,w,i-1/2}^{n+1} T_{w,i-1/2}^{n+1}}{\Delta \zeta} \\ &+ \frac{1}{(R_0 - R)} \frac{1}{\Delta \zeta} \left[\lambda_{w,i+1/2}^{n+1} \left(\frac{T_{w,i+1}^{n+1} - T_{w,i}^{n+1}}{(R_0 - R) \Delta \zeta} \right) \right. \\ &\quad \left. - \lambda_{w,i-1/2}^{n+1} \left(\frac{T_{w,i}^{n+1} - T_{w,i-1}^{n+1}}{(R_0 - R) \Delta \zeta} \right) \right] \\ &- \frac{1}{R_0 - R} \frac{[(c_{pw} f_w)_i T_{w,i}^{n+1} - (c_{pw} f_w)_{i-1} T_{w,i-1}^{n+1}]}{\Delta \zeta} \end{aligned} \quad (31)$$

where n is the current iteration index and $n + 1$ is the new iteration index.

Solidification interface equation:

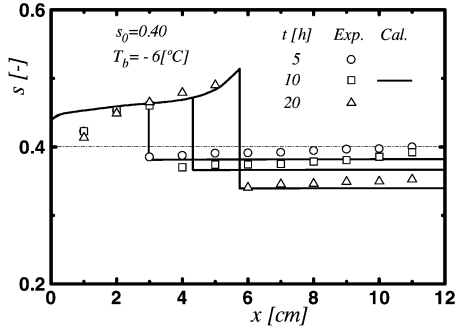
$$\begin{aligned} &\rho_i L \varepsilon s_{ib} \frac{3R^{n+1} - 4R^n - R^{n-1}}{2\Delta t} \\ &= \lambda_i \frac{3T_{i,end}^{n+1} - 4T_{i,end-1}^{n+1} + T_{i,end-2}^{n+1}}{2R^{n+1} \Delta \eta} \Big|_{x=R} \\ &- \lambda_w \frac{-3T_{w,0}^{n+1} + 4T_{w,1}^{n+1} - T_{w,2}^{n+1}}{2(R_0 - R^{n+1}) \Delta \zeta} \Big|_{x=R} \end{aligned} \quad (32)$$

Finally, to obtain values for all stage variables at the marching time, the system of nonlinear equations must be resolved. This procedure is carried out in two distinct stages, including outer and inner iteration phases. During the outer iteration phase, the system of the nonlinear equations is linearized according to the standard Newton–Raphson method. During the inner iteration phase, the system is solved by employing the standard successive overrelaxation method.

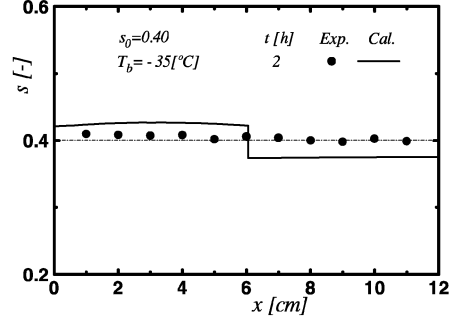
VII. Results and Discussion

The predicted results for one-dimensional solidification in the unsaturated granular packed bed are compared with the experimental results. Typical ice saturation and water saturation profiles for different initial water saturations and different cooling temperatures are shown in Fig. 8.

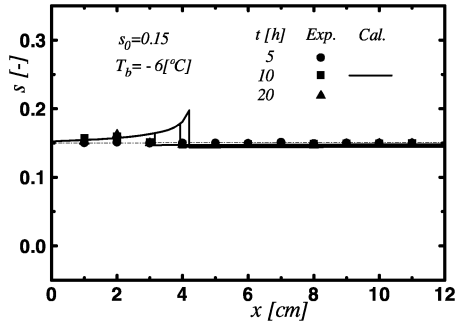
Figure 8a shows the saturation profiles in the porous packed bed for the case of $s_0 = 0.4$ and $T_b = -6^\circ\text{C}$ as a parameter of elapsed time. According to the explanation in Sec. III, the ice saturation in the frozen layer is increased, while the water saturation in the unfrozen layer is decreased in comparison with the initial state. It is found that the water transport due to capillary action plays an important role for the solidification process. As the solidification progresses, the freezing heat flux will be decreased due to the increasing of thermal resistance of the frozen layer. Accordingly, in Fig. 8a a



a)



b)



c)

Fig. 8 Predicted and experimental profiles for ice and water saturations at various conditions: a) $s_0 = 0.4$ and $T_b = -6^\circ\text{C}$, b) $s_0 = 0.4$ and $T_b = -35^\circ\text{C}$, and c) $s_0 = 0.15$ and $T_b = -6^\circ\text{C}$.

location of lower freezing heat flux ($x = 5.8$ cm) results in a higher ice saturation, in spite of the lower rate of the absorption of water, as referred to Fig. 5. This is because the freezing heat flux affects both the rates of the absorption of water and the motion of frozen layer. Specifically, a lower freezing heat flux leads to a lower rate of absorption of water and a slower progressing rate of the frozen layer at the same time. Nevertheless, depending on the freezing heat flux these two rates have a contrary effect on ice saturation at the solidification interface. As the freezing heat flux is increasing, particularly at the early stages of the solidification process (about 5 h), the progressing rate of the frozen layer is superior to the rate of the absorption of water and the ice saturation is decreasing. The calculated results of ice and water saturations in the packed bed are in agreement with the experimental results for solidification process.

Figures 8b and 8c show the saturation profiles as a parameter of elapsed time for the case of $s_0 = 0.4$, $T_b = -35^\circ\text{C}$ and $s_0 = 0.15$, $T_b = -6^\circ\text{C}$, respectively. It is evident from Figs. 8b and 8c that the profiles of ice and water saturation in both cases become almost uniform over the packed bed. This is because the water transport does not play an important role on the solidification process. Comparing with the case of Fig. 8a, we can see that the freezing heat flux is very high in the case of Fig. 8b, whereas a lower rate of water absorption occurs in the case shown in Fig. 8c. This is because the progressing rate of the frozen layer is superior to the rate of water absorption

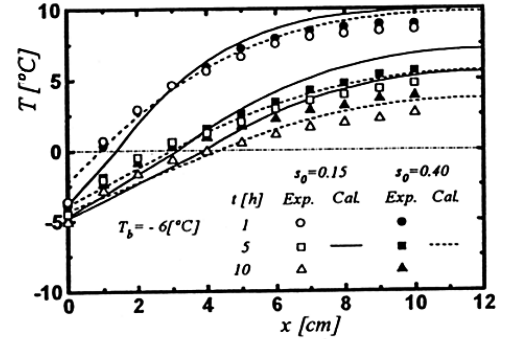


Fig. 9 Comparison between the predicted and experimental temperature profiles for the cases of $s_0 = 0.15$ and $s_0 = 0.4$.

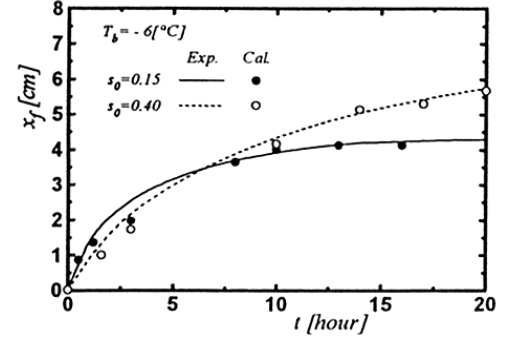


Fig. 10 Predicted and experimental solidification interface for the cases of $s_0 = 0.15$ and $s_0 = 0.4$.

and the effect of the water transport due to the solidification process is not remarkable.

Next, Fig. 9 shows the temperature profiles in the packed bed for the case of $s_0 = 0.4$ and $s_0 = 0.15$ with $T_b = -6^\circ\text{C}$ as a parameter of elapsed time. In general, the temperature in the packed bed drops faster in the case of $s_0 = 0.4$ compared with the case of $s_0 = 0.15$. This is because the thermal conductivity in frozen layer becomes higher in the former case as a result of higher ice saturation.

Figure 10 shows the time variation of the frozen layer thickness for the case $s_0 = 0.4$ and $s_0 = 0.15$ with $T_b = -6^\circ\text{C}$. It is observed that the thickness of the case of $s_0 = 0.4$ becomes thinner in the early stages of the solidification process, but after that it becomes thicker compared with that of $s_0 = 0.15$. These results may be explained by considering the latent heat of freezing and thermal conductivity in the frozen layer. In general, a higher ice saturation corresponding to the case of $s_0 = 0.4$ needs more latent heat per unit volume of the layer and at the same time leads to a higher thermal conductivity. That is, in the early stages of the solidification process, a higher ice saturation results in a thinner thickness in comparison to the case of $s_0 = 0.15$ due to the effect of latent heat of freezing as explained earlier. However, at the long stages of the solidification process, the thickness becomes thicker due to the higher thermal conductivity. The calculated results are in agreement with the experimental results for the solidification process.

VIII. Conclusions

The solidification process in an unsaturated granular packed bed has been investigated theoretically and numerically, by the consideration of water transport toward the solidification interface due to capillary action. It is found that the rate of the absorption of water in the frozen layer due to the solidification is related to the freezing heat flux and the water saturation at the solidification interface. Also, it is found that the ice saturation in the frozen layer becomes higher for a higher rate of water absorption and lower freezing heat flux.

The one-dimensional solidification model associated with phase-change conditions and the coordinate transformation techniques for

a moving boundary problem are completely presented to calculate the rate of absorption of water in the frozen layer, the temperature and water saturation distributions, and the interface position. It is found that the predicted results are in agreement with the experimental results.

Acknowledgment

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